THE EFFECT OF MOTION ENERGY DISSIPATION ON HEAT TRANSFER FOR LAMINAR FLOW OF NEWTONIAN FLUIDS IN CIRCULAR PIPES

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An analytic solution is obtained for the problem of heat transfer in the laminar flow of Newtonian fluids in circular pipes taking account of motion energy dissipation.

In many engineering applications it is extremely important to take account of the effect of dissipative heating on heat transfer in the laminar flow of Newtonian fluids in pipes. But up to the present time the study of these problems has not been sufficiently explicit; in particular there are no data which make it possible to give not only a qualitative but also a quantitative estimate of the effect of the dissipative factor on the fundamental characteristics of heat transfer.

The aim of this paper is to fill this gap to some extent. We consider heat transfer in the steady motion of Newtonian fluids in a circular pipe when the friction curves can be approximated by a rheological power law

$$\tau = k\dot{\gamma}^n. \tag{1}$$

When the velocity profile is established, the heat flux along the axis of the pipe is small by comparison with the heat flux radially, the physical properties of the fluid are constant and there are no additional internal heat sources, the energy equation can be written

$$\rho c_p u \frac{\partial T}{\partial x} = \frac{\lambda}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \Phi_{\mathrm{d}}(r). \tag{2}$$

From the equation of motion, neglecting body forces, and taking note of [1], we obtain a nondimensional expression for the velocity distribution and the dissipation functions:

$$u(\eta) = \frac{m+3}{m+1} V(1-\eta^{m+1}), \tag{3}$$

$$\Phi_{\mathrm{d}}(\eta) = \frac{m+3}{2} W\eta^{m+1}. \tag{4}$$

In nondimensional coordinates Eq. (2) has the form

$$(1-\eta^{m+1})\frac{\partial\vartheta}{\partial\xi} = \frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\vartheta}{\partial\eta}\right) + \frac{m+3}{2}\eta^{m+1} \tag{5}$$

with the boundary conditions

$$\vartheta(0,\ \eta) = \vartheta_0;\quad \vartheta(\xi,\ 1) = 0;\quad \frac{\partial\vartheta(\xi,\ 0)}{\partial\eta} = 0. \tag{6}$$

This problem was considered in the above formulation in [1] for the particular case when the fluid temperature at the beginning of the pipe was equal to the wall temperature. In addition a solution was obtained in a form which prevented precise clarification of the effect of dissipation energy on the fundamental characteristics of heat transfer. The general solution only gave finite results for n = 0.5 (m = 2).

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TABLE 1. Values of Nu_∞ for Various m

m	Nu_∞ ($\beta=0$)	Nu_∞ ($\beta\neq 0$)	m	Nu_∞ ($\beta=0$)	Nu_∞ ($\beta\neq 0$)
1	3,6567	9,6000	6	4,6192	19,8000
2	3,9494	11,6667	7	4,7193	21,8182
3	4,1753	13,7143	8	4,8044	23,8333
4	4,3544	15,7500	9	4,8771	25,8461
5	4,4995	17,7778	10	4,9402	27,8571

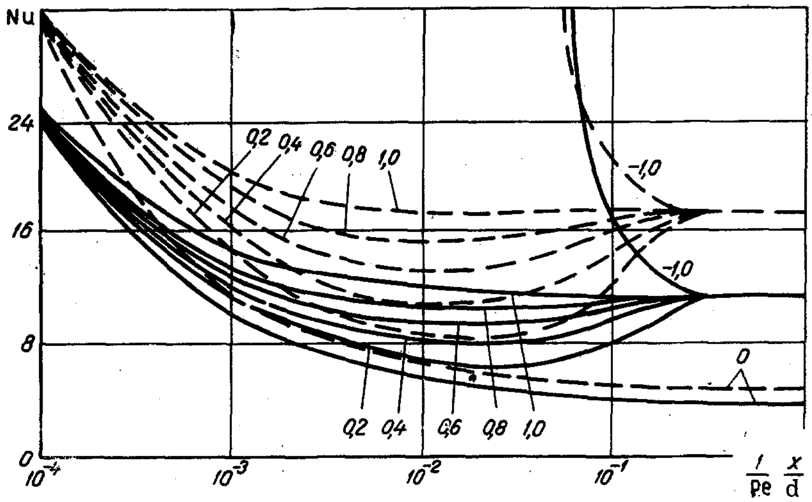

Fig. 1. The local Nusselt number as a function of the pipe length [continuous lines) m = 2; dotted lines) 5. The figures attached to the curves are the values of β].

An acceptable method of solution was given in [2]. Using the eigenvalues and eigenfunctions given in [3] the author has qualitatively analyzed the effect of internal heat generation for n = 1/3.

In neither [1] or [2] was the most interesting region near to the beginning of the pipe investigated, since when the number of eigenvalues is restricted (in [3] only the first three eigenvalues were given) large errors occur in the solution for this region.

A larger region was considered in [4] but heat transfer was only discussed for a uniform distribution of internal heat sources across a section of the flow.

We seek a solution of Eq. (5) as the sum of two functions

$$\vartheta(\xi, \eta) = \vartheta_1(\eta) + \vartheta_2(\xi, \eta), \tag{7}$$

where $\vartheta_1(\eta)$ is the temperature in a stabilized segment of heat transfer satisfying the equation

$$\frac{1}{\eta}\frac{d}{d\eta}\left(\eta\frac{d\vartheta_1}{d\eta}\right) + \frac{m+3}{2}\eta^{m+1} = 0 \tag{8}$$

with the boundary conditions

$$\left.\frac{d\vartheta_1}{d\eta}\right|_{\eta=0} = \left.\vartheta_1\right|_{\eta=1} = 0.$$

The function $\vartheta_2(\xi, \eta)$ satisfies the equation

$$(1-\eta^{m+1})\frac{\partial\vartheta_2}{\partial\xi} = \frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\vartheta_2}{\partial\eta}\right) \tag{9}$$

with the boundary conditions

$$\vartheta_2(0, \eta) = \vartheta_0 - \vartheta_1; \quad \vartheta_2(\xi, 1) = \frac{\partial\vartheta_2(\xi, 0)}{\partial\eta} = 0.$$

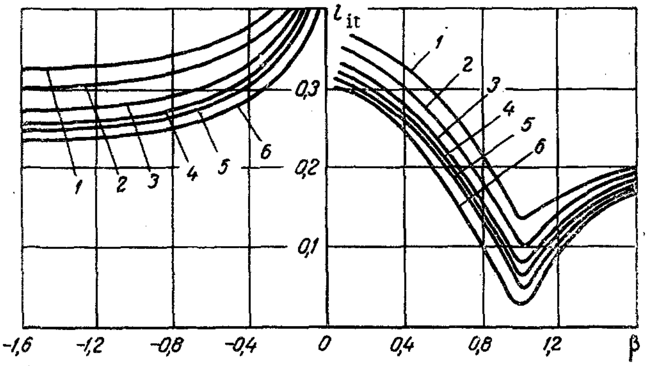

Fig. 2. The length of the initial thermal segment l_{it} as a function of the parameter β [1) m = 1; 2) 2; 3) 4; 4) 6; 5) 8; 6) 10].

We seek the solution of Eq. (9) by Fourier's method of expanding in a series of eigenfunctions of $\Phi(\eta)$ satisfying the Sturm–Liouville equation.

We can write the general solution of Eq. (5) as

$$\theta = \frac{T - T_w}{T_0 - T_w} = \sum_{k=0}^{\infty} (A_k^0 - \beta A_k^*)\, \Phi_k(\eta) \exp\left(-4c_k^2 \frac{m+1}{m+3} \frac{1}{\mathrm{Pe}} \frac{x}{d}\right) + \beta(1 - \eta^{m+3}), \tag{10}$$

where the coefficients A_k^0 and A_k^* are defined by the equations

$$A_k^0 = \frac{\int_0^1 \Phi_k(\eta)\, \eta\, (1 - \eta^{m+1})\, d\eta}{\int_0^1 \Phi_k^2(\eta)\, \eta\, (1 - \eta^{m+1})\, d\eta},$$

$$A_k^* = \frac{\int_0^1 \Phi_k(\eta)\, \eta\, (1 - \eta^{m+1})(1 - \eta^{m+3})\, d\eta}{\int_0^1 \Phi_k^2(\eta)\, \eta\, (1 - \eta^{m+1})\, d\eta}$$

The coefficient β is a nondimensional parameter taking account of the effect of mechanical energy dissipation on heat transfer,

$$\beta = \frac{d^2 W}{2(m+3)\lambda(T_0 - T_w)}. \tag{11}$$

We can write β in a more convenient form

$$\beta = \frac{4}{m+3} \frac{k_m' \, 8^{\frac{1}{m} - 1} d^{1 - \frac{1}{m}} V^{1 + \frac{1}{m}}}{\lambda(T_0 - T_w)} \tag{12}$$

for a Newtonian fluid β is the product of the Eckert and Prandtl numbers. For a non-Newtonian fluid we can obtain a similar product with a constant factor if we represent the generalized Prandtl number in the Metzner and Reed [5] form.

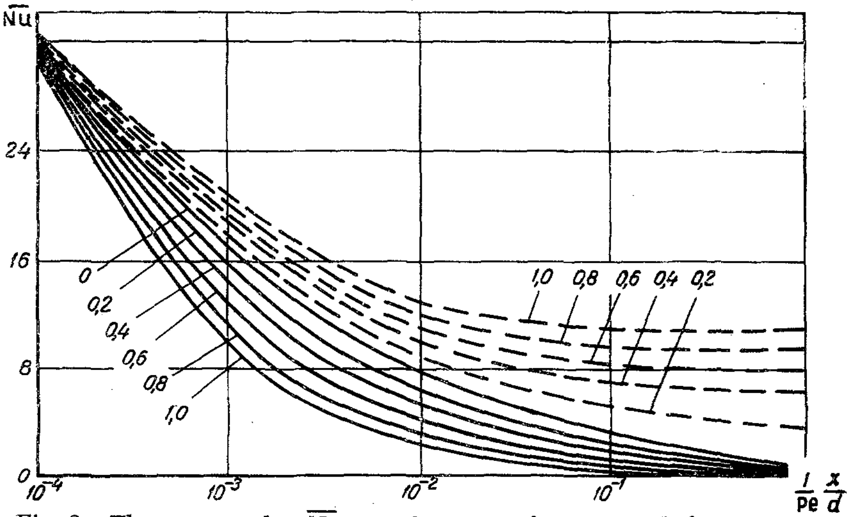

Fig. 3. The mean value $\overline{\mathrm{Nu}}$ as a function of pipe length for m = 2 [continuous lines) using Eq. (16); dotted lines) using Eq. (18); the numbers attached to the graphs are the values of β].

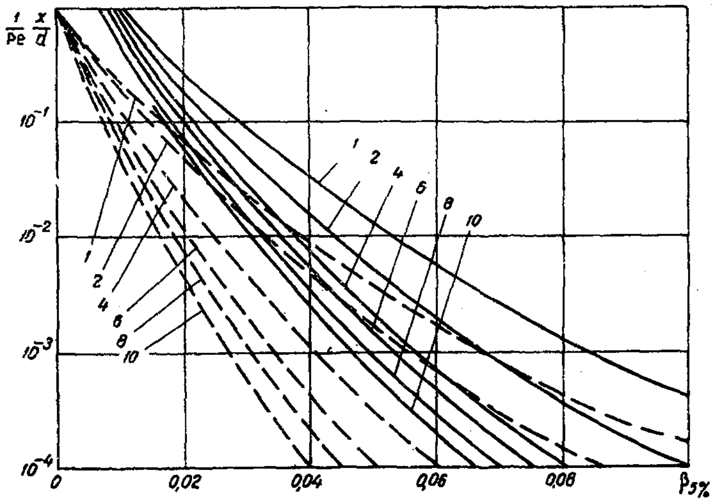

Fig. 4. Limiting values of the parameter $\beta_{5\%}$, taking account of energy dissipation [continuous lines) using Eq. (16); dotted lines) using Eq. (18); the figures attached to the curves are the values of m].

The reduced mean mass temperature at a given cross-section of the pipe is determined by the equation

$$\bar{\theta} = \frac{\bar{T} - T_w}{T_0 - T_w} = \beta \frac{m+4}{m+5} + 4\frac{m+3}{m+1} \sum_{k=0}^{\infty} \frac{B_k^0 - \beta B_k^*}{c_k^2} \times \exp\left(-4\frac{m+1}{m+3} c_k^2 \frac{1}{\mathrm{Pe}} \frac{x}{d}\right), \tag{13}$$

where

$$B_k = -\frac{1}{2} A_k \left(\frac{d\Phi_k}{d\eta}\right)_{\eta=1}.$$

Having defined the local heat-transfer coefficient referred to the difference between the mean mass temperature of the fluid and the wall temperature, we can obtain an expression for the local Nusselt number:

$$\mathrm{Nu} = \left\{\beta \frac{m+3}{2} + \sum_{k=0}^{\infty} (B_k^0 - \beta B_k^*) \exp\left(-4\frac{m+1}{m+3} c_k^2 \frac{1}{\mathrm{Pe}} \frac{x}{d}\right)\right\}$$
$$\times \left\{\beta \frac{m+4}{4(m+3)} + \frac{m+3}{m+1} \sum_{k=0}^{\infty} \frac{B_k^0 - \beta B_k^*}{c_k^2} \times \exp\left(-4\frac{m+1}{m+3} c_k^2 \frac{1}{\mathrm{Pe}} \frac{x}{d}\right)\right\}^{-1} \tag{14}$$

It follows from (14) that the limiting Nusselt number is

$$\mathrm{Nu}_\infty = \frac{m+1}{m+3} c_0^2 \quad \text{for} \quad \beta = 0,$$
$$\mathrm{Nu}_\infty = 2\frac{(m+3)(m+5)}{m+4} \quad \text{for} \quad \beta \neq 0. \tag{15}$$

Values of Nu_∞ for m = 1-10 are given in Table 1. For $\beta = 0$ these values agree well with the results of [6].

Using Eqs. (10), (13), and (14) a Minsk-22 computer was programmed to calculate the temperature distribution at various cross-sections of the pipe, the change in the mean mass temperature distribution and the local Nusselt number for reduced pipe lengths $(1/\mathrm{Pe})(x/d) = 1 \cdot 10^{-4}$-1 and the values of m given in Table 1.

For the above range of variations in the reduced lengths at the upper boundary $((1/\mathrm{Pe})(x/d) = 1 \cdot 10^{-4})$ reliable results were obtained with nine eigenfunctions and eigenvalues which were defined for all values of m. Since there are no similar results in the literature, data for a Newtonian fluid (m = 1) [7] were used for verification. Satisfactory agreement was also obtained for available data on non-Newtonian fluids [3, 4].

The Nusselt number is shown on Fig. 1 as a function of the reduced length for m = 2 and 5. From a general evaluation of the results we have obtained we can note the increased effect of dissipative heating on heat transfer as m increases. Hence the role of dissipative heating for non-Newtonian fluids is always greater when β = const than for Newtonian fluids.

The extent of this influence increases when m = const as the reduced length increases; all the curves approach a limiting value independent of β. This is a logical contradiction of the nature of the curve when $\beta = 0$, the limiting value of which is several times less than Nu_∞ when $\beta = 0$.

Since this situation, which to some extent we knew, has not been studied for a Newtonian fluid, special investigation of the problem was made. It appears that Nu_∞ decreases as β decreases, tending to a limiting value Nu_∞ for $\beta = 0$, but this reduction begins for very small values of $\beta \leq 1 \cdot 10^{-6}$, the effect of which in the initial stages is in general negligible. The behavior of the function Nu = f((1/Pe)(x/d)) is different for positive (cooling) and negative (heating) values of β.

For $\beta > 0$ and $0 < \beta < 1$, initially Nu decreases along the pipe and then increases to Nu_∞; when $\beta > 1$, Nu continuously decreases along the length of the pipe. The value of β at which the nature of the function changes depends very weakly on m, decreasing slightly as m increases.

When $\beta < 0$, the function is discontinuous, the upper branch of the curve, going off to infinity, corresponding to a change in the direction of the flux.

Energy dissipation also has a significant influence on the length of the initial thermal segment. Figure 2 shows the reduced length of the initial thermal segment for which Nu, to within 1%, is equal to Nu_∞ for m = 1-10, as a function of β. The reduction in Nu as β increases, when $\beta > 0$, which as been noted for Newtonian fluids [8] holds, as we see only when β varies within definite limits. The minimum of the curves lies in the region $\beta \approx 1$ and corresponds to a change in the nature of the function Nu = f((1/Pe)(x/d)).

When $\beta < 0$, the reduced length l_{it} monotonically decreases as β increases.

For very small values of β $(1 \cdot 10^{-6})$ the curve approaches values of the reduced length when $\beta = 0$ (0.055-0.05 for m = 1-10); on Fig. 2 this bend coincides with the ordinate axis and conventionally is not shown.

We define the mean value $\overline{Nu}$ from the heat-transfer coefficient referred to the arithmetic mean of the temperature distribution. If in addition the quantity of heat absorbed by the fluid is defined in the usual way from the difference between the temperatures at the beginning of the pipe and at a given section, then the expression for $\overline{Nu}$ takes the form

$$\overline{Nu} = \frac{1}{2}\,\mathrm{Pe}\,\frac{d}{x}\,\frac{1-\overline{\theta}}{1+\overline{\theta}} \tag{16}$$

with a similar expression for large values of the reduced length, when $\overline{\theta}$ = const,

$$\overline{Nu}_\infty = \frac{1}{2}\,\mathrm{Pe}\,\frac{d}{x}\,\frac{(m+5)-\beta(m+4)}{(m+5)+\beta(m+4)}\,. \tag{17}$$

Equation (16) is suitable for engineering calculations of the heat transfer inside the pipe, but does not correspond to the actual quantity of heat absorbed by the heat transfer surface: it is reduced by heat dissipation on cooling and increased by it on heating.

The value of $\overline{Nu}$, computed from the actual heat flux through the wall, is

$$\overline{Nu}_d = \frac{1}{2}\,\mathrm{Pe}\,\frac{d}{x}\,\frac{1-\overline{\theta}}{1+\overline{\theta}} + 4\beta(m+3)\frac{1}{1+\overline{\theta}}\,. \tag{18}$$

The heat transfer for the external medium is calculated from (18).

Figure 3 gives the results of calculating $\overline{Nu}$ and $\overline{Nu}_d$ from (16) and (18) for m = 2 and $\beta > 0$. As the dissipative factor increases, $\overline{Nu}$ decreases and $\overline{Nu}_d$ increases. Thus in spite of the increase in the actual heat-transfer coefficient, the intensity of cooling continuously decreases as β increases. In the limit at some distance from the beginning of the pipe ($\overline{Nu} = 0$ and correspondingly $\overline{\theta} = 1$) the mean mass temperature is equal to the fluid temperature at the beginning of the pipe. Then $\overline{Nu}_\infty$ is negative, which corresponds to values $\beta > 1$ (Eq. (17)), being smaller, the larger m is. For sufficiently large but real values of $\beta > 0.7$, $\overline{\theta}$

initially decreases and then stabilizes. Obviously a further increase in the reduced length leads to a sharp reduction in the cooling effect. Thus by analyzing Eqs. (13) and (16) we can solve a very important engineering problem – the choice of the optimum pipe length in heat-transfer apparatuses designed for cooling high-viscosity non-Newtonian fluids.

As we see from the foregoing, taking account of the dissipative factor in solving laminar heat-transfer problems leads to an increase in the complexity of the computational expressions. Hence those values of β are of interest, for which the effect of this factor can be neglected for a given accuracy of calculations. Figure 4 shows such values, each curve bounding the limiting values of $\beta_{5\%}$ giving an error of not more than 5% in the determination of $\overline{Nu}$ and $\overline{Nu}_d$, if the equations in which the dissipative factor is ignored are used for the calculation.

As the reduced length and m increase, the limiting values of β decrease; the nature of the effect of m completely corresponds to the above conclusion about the increase the role of the dissipative factor in laminar heat transfer in non-Newtonian fluids by comparison with that in Newtonian fluids.

The equations given above for the effect of dissipative heating were obtained under the condition that the physical properties of the fluid were independent of the temperature. The problem actually arises to what extent this assumption (it principally effects the rheological parameters) effects the final results.

In the solutions known to us of the nonisothermal problem, the dissipative factor was ignored [9, 10, 11] and so the problem which interests us cannot be clarified by comparison with results in the literature.

In this connection, to make an approximate evaluation of the error in the results we obtained, we carried out a number of numerical calculations of the nonisothermal problem by successive approximations for high-viscosity non-Newtonian fluids (plastic lubricants), the rheological properties of which were given in [12].

The flow curves for these fluids, within the sheer stress limits τ_w–$0.25\tau_w$, were approximated by a power law using Eq. (1) with the introduction of a temperature factor, the effect of which on the index of the non-Newtonian behavior can be ignored for the measured temperature drops.

Analysis of the results obtained showed that the qualitative side of the problem is reflected quite correctly by the foregoing solutions. However, the following considerations apply to the quantitative results.

For small values of the reduced length the temperature drops were considerable at sections of the flow but the effect of the dissipative factor was small. As the reduced length increased the temperature profile became more uniform and at the same time the role of the dissipative factor increased. Obviously there must be a particular region of reduced lengths in which the errors in the solution are approximately the same. From the results we obtained this interval is bounded by the reduced length values $10^{-4} < (1/\mathrm{Pe})(x/d) < 5\cdot 10^{-3}$ and includes the most important region of solutions for engineering practice.

When $\beta < 0.8$ and $(k'_w/k'_m) < 2.5$–3.0 the error in the results in this region does not exceed ±15% (positive values of the errors refer to cooling, $\beta > 0$; negative values refer to heating $\beta < 0$), if the nonisothermal conditions are taken into account by introducing the Eider and Tate correction, modified by Metzner for non-Newtonian fluids [13]. The physical constants in the dissipation parameter (Eq. (12)), are determined from the mean mass temperature of the fluid.

NOTATION

τ	is the shear stress;
$\dot{\gamma}$	is the shear rate;
$\eta = r/r_w$	is the dimensionless coordinate;
V	is the mean fluid velocity;
W	is the density of internal heat sources per unit volume;
$\Phi_d(\eta)$	is the dissipative factor;
$\vartheta = (T - T_w)/(Wr_w^2/\lambda)$;	
$\theta = (T - T_w)/(T_0 - T_w)$	is the reduced nondimensional temperature;
c_k	are the eigenvalues;
$\Phi_k(\eta)$	are the eigenfunctions;
$k' = k((3n + 1)/4n)^n$	is the constant in the rheological equation $\tau = k'(4\Gamma)^{n'}$;
Γ	is the mean shear rate;

d		is the pipe diameter;
λ		is the thermal conductivity of the fluid;
$m = 1/n$, where $n = n'$		is the index of non-Newtonian behavior in (1).

Subscripts

w refers to the wall;
m refers to the mean mass temperature;
0 refers to the entrance.

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